

CBCS SCHEME

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15MATDIP31

Third Semester B.E. Degree Examination, June/July 2023 Additional Mathematics – I

Time: 3 hrs.

Max. Marks: 80

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

1 a. Express the complex number $\frac{1}{(2+i)^2}$ in the form $a + ib$. (05 Marks)

b. Simplify $\frac{(\cos\theta - i\sin\theta)^2(\cos 7\theta + i\sin 7\theta)^{-3}}{(\cos 4\theta - i\sin 4\theta)^9(\cos\theta + i\sin\theta)^5}$. (05 Marks)

c. Find the value of λ such that the vectors $(1, -2, 3)$, $(-2, 3, -4)$ and $(1, -3, \lambda)$ are coplanar. (06 Marks)

OR

2 a. Find the modulus and amplitude of the complex number $1 + i\sqrt{3}$. (05 Marks)

b. If $\vec{a} = \hat{i} + \hat{j} + 2\hat{k}$ and $\vec{b} = -\hat{i} + 2\hat{j} - \hat{k}$ find

i) $(\vec{a} + \vec{b}) \cdot (\vec{a} - \vec{b})$

ii) $(\vec{a} + \vec{b}) \times (\vec{a} - \vec{b})$. (05 Marks)

c. If $\vec{a} = (1, -1, 2)$, $\vec{b} = (1, 2, 3)$ and $\vec{c} = (3, -2, 4)$. Evaluate $\vec{a} \times (\vec{b} \times \vec{c})$ and $(\vec{a} \times \vec{b}) \times \vec{c}$. Are they equal? (06 Marks)

Module-2

3 a. If $y = A \cos(\log x) + B \sin(\log x)$, show that $x^2 y_{n+2} + (2n+1)xy_{n+1} + (x^2 + 1)y = 0$. (05 Marks)

b. Expand $\log(1+x)$ in a Maclaurin's series upto the term involving x^4 . (05 Marks)

c. If $u = \sin^{-1} \left[\frac{x^4 + y^4}{x+y} \right]$, show that $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = 3 \tan u$. (06 Marks)

OR

4 a. Write the n^{th} derivatives of

i) $\frac{1}{2x+3}$

ii) $\log(3x+2)$

iii) $e^{2x} \cos 3x$. (05 Marks)

b. Find the pedal equation of $r^n = a^n \cos n\theta$. (05 Marks)

c. If $x = r \cos\theta$, $y = r \sin\theta$, $z = z$ then find $\frac{\partial(x, y, z)}{\partial(r, \theta, z)}$. (06 Marks)

Module-3

5 a. Evaluate

i) $\int_0^{\pi/2} \sin^6 x \, dx$

ii) $\int_0^{\pi/2} \sin^4 x \cos^5 x \, dx.$

(05 Marks)

b. Evaluate $\int_0^{2a} \int_0^{x^2/4a} xy \, dy \, dx.$

(05 Marks)

c. Evaluate $\int_{-1}^1 \int_0^z \int_{x-z}^{x+z} (x+y+z) \, dy \, dx \, dz.$

(06 Marks)

OR

6 a. Evaluate $\int_0^{\pi/6} \cos^4 3\theta \sin^3 6\theta \, d\theta.$

(05 Marks)

b. Evaluate $\int_1^2 \int_1^3 xy^2 \, dx \, dy.$

(05 Marks)

c. Evaluate $\int_{-c}^c \int_{-b}^b \int_{-a}^a (x^2 + y^2 + z^2) \, dx \, dy \, dz.$

(06 Marks)

Module-47 a. A particle moves along a curve $x = e^t$, $y = 2\cos 3t$, $z = 2\sin 3t$, where t is the time variable. Determine the magnitudes of velocity and acceleration at $t = 0$. (05 Marks)b. If $\phi = \log(x^2 + y^2 + z^2)$, find the magnitude of the grad ϕ at $(1, 2, 3)$. (05 Marks)c. If $\vec{R} = x\hat{i} + y\hat{j} + z\hat{k}$ find $\nabla \cdot \vec{R}$ and $\nabla \times \vec{R}$. (06 Marks)

OR

8 a. Find a unit vector normal to the surface $xy^3z^2 = 4$ at $(-1, -1, 2)$. (05 Marks)b. Find the value of 'a' if the vector $(ax^2y + yz)\hat{i} + (xy^2 - xz^2)\hat{j} + (2xyz - 2x^2y^2)\hat{k}$ is solenoidal. (05 Marks)c. Show that grad $(x^3 + y^3 + z^3 - 3xyz)$ is irrotational. (06 Marks)Module-5

9 a. Solve $\frac{dy}{dx} = \frac{y}{x} + \sin\left(\frac{y}{x}\right).$

(05 Marks)

b. Solve $\frac{dy}{dx} - \frac{y}{x} = (x+1).$

(05 Marks)

c. Solve $(x^2 + y^2 + 1) \, dx + 2xy \, dy = 0.$

(06 Marks)

OR

10 a. Solve $(3x^2 + 6xy^2) \, dx + (6x^2y + 4y^3) \, dy = 0.$

(05 Marks)

b. Solve $(xy^3 + y) \, dx + 2(x^2y^2 + x + y^4) \, dy = 0.$

(05 Marks)

c. Solve $\frac{dy}{dx} = y \tan x - y^2 \sec x$

(06 Marks)
